EL21 Lab 1 live

Thursday, January 21, 2021 9:50 AM

ODE's ! Today: • linear first order • separable • Exact

1. $\frac{dy}{dx} = \frac{x^2}{1-y^2} = (x^2)\cdot\left(\frac{1}{1-y^2}\right)$ $\frac{dy}{dx} = g(x)\cdot h(y)$

2. $\frac{dy}{dx} = \frac{x^2 - x^3y}{(1-xy)(1+y)} = \frac{x^2\cancel{(1-xy)}}{\cancel{(1-xy)}(1+y)} = \frac{x^2}{1+y} = (x^2)\left(\frac{1}{1+y}\right)$

3. $\frac{x}{x}\frac{dy}{dx} + \frac{x^2}{x}y = \frac{xe^x}{x}$

Standard form: $1\cdot\frac{dy}{dx} + xy = e^x.$

4. $\underbrace{(2x+3)}_{M(x,y)}dx + \underbrace{(2y-2)}_{N(x,y)}dy = 0$ $\frac{\partial}{\partial y}$ $\frac{\partial}{\partial x}$

Exact if: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Exact!

$0 = 0$

5. $\frac{dy}{dx} = \frac{-(y\cos(x) + 2xe^y)}{(\sin(x) + x^2e^y - 1)}$

Rewrite: $\underbrace{(y\cos(x) + 2xe^y)}_{M(x,y)}dx + \underbrace{(\sin(x) + x^2e^y - 1)}_{N(x,y)}dy$

$\frac{\partial M}{\partial y} = \cos(x) + 2xe^y$; $\frac{\partial N}{\partial x} = \cos(x) + 2xe^y$

Exact!

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1. $\frac{dy}{dx} = \frac{x^2}{1-y^2}$; $\boxed{y(0) = 1.}$

Rearrange: $(1-y^2)dy = x^2dx$

Integrate: $\int(1-y^2)dy = \int x^2dx$

$\left(y - \frac{y^3}{3}\right) = \frac{x^3}{3} + C$

$\boxed{y\left(1 - \frac{y^2}{3}\right) = \frac{x^3}{3} + C}$

Implicit Solution! Cannot solve as $y(x) = \sim\sim$

Solve for C: $y(0)\left(1 - \frac{y^2(0)}{3}\right) = \frac{0^3}{3} + C$

$1\cdot\left(1 - \frac{1}{3}\right) = C$

$\Rightarrow C = 2/3$

Final Solution: $\boxed{y\left(1 - \frac{y^2}{3}\right) = \frac{x^3}{3} + \frac{2}{3}}$

3. $1\cdot\frac{dy}{dx} + x\underset{g(x)}{y} = \underset{f(x)}{e^x}$; ~~y(x) = ln(x)~~ (Int. factor)

i.) Standard form !!!

ii.) Int. factor : $\mu(x) = e^{\int g(x)dx}$, $g(x) = x$.

$\boxed{\mu(x) = e^{\int x dx} = e^{x^2/2}}$. $\longrightarrow$ Can also use $y(x) = \frac{1}{\mu(x)}\int \mu(x) f(x)dx$

iii.) With the int. factor found, we can multiply the whole equation by $\mu(x)$:

$e^{x^2/2}\frac{dy}{dx} + xe^{x^2/2}y = e^x\cdot e^{x^2/2}$

$= \frac{d}{dx}(\mu(x)y(x))$

$= \frac{d}{dx}\left(e^{x^2/2}\cdot y\right) = e^{x + x^2/2}$

iv.) Integrate !

$\int\frac{d}{dx}\left(e^{x^2/2}y\right) = \int e^{x + x^2/2}dx$

$e^{x^2/2}y = \int e^{x+x^2/2}dx + C$

$\boxed{y(x) = e^{-x^2/2}\int e^{x+x^2/2}dx + Ce^{-x^2/2}}$

??

$\underbrace{(2x+3)}_{M(x,y)}dx + \underbrace{(2y-2)}_{N(x,y)}dy = 0$

Idea: Find an implicit solution $\psi(x,y) = C$

Using: $\begin{cases}\psi_x = M(x,y)\\ \psi_y = N(x,y)\end{cases}$ $\quad \psi_x = \frac{\partial\psi}{\partial x}$, $\psi_y = \frac{\partial\psi}{\partial y}$

i.) Show it is exact!

ii.) Set up equations: $\begin{cases}\psi_x = 2x+3\\ \psi_y = 2y-2\end{cases}$

Choose one and integrate! (we'll take the first)

$\psi(x,y) = \int\psi_x dx = \int(2x+3)dx$
$= x^2 + 3x + h(y)$

iii.) Solve for $h(y)$ using the other equation!

$\psi(x,y) = x^2 + 3x + h(y)$

$\frac{\partial\psi}{\partial y} = h'(y) = 2y-2$

$\int\frac{dh}{dy}dy = \int(2y-2)dy$

$h(y) = y^2 - 2y$

Therefore, $\psi(x,y) = \boxed{x^2 + 3x + y^2 - 2y = C}$

If given an initial condition: $y(0) = 4.$

$\psi(0, y(0)) = 0^2 + 3(0) + y(0)^2 - 2y(0) = C$
$= 4^2 - 2\cdot 4 = C$
$= 16 - 8 = 8 = C$

$\Rightarrow \boxed{C = 8}$

Final Solution: $\psi(x,y) = 8$, or

$\boxed{y^2 - 2y + x^2 + 3x - 8 = 0}$ Implicit!