EL18 Lab 10 - live

Tuesday, November 17, 2020 5:59 PM

Line Integrals & vector fields

Given a vector field F, F is called conservative if $F = \nabla f$, for some function f.

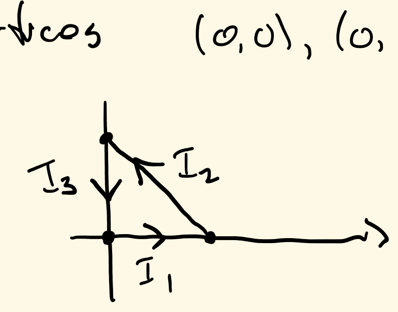
$$\langle P, Q \rangle = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

How to check: look at $\frac{\partial P}{\partial y}$ and $\frac{\partial Q}{\partial x}$.

If $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$, then $F = \langle P, Q \rangle$ is conservative.

Otherwise, it is not conservative.

Ex. Evaluate $\int_C y^3 dx - x^3 dy$, where C is the triangle with vertices $(0,0), (0,1), (1,0)$ oriented counter-clockwise.

Picture: 

$\rightsquigarrow$ use that $\int_C = \int_{I_1} + \int_{I_2} + \int_{I_3}$.

First, parametrize the curves!

I_1: $y = 0$, $0 \le x \le 1$. So, $I_1 = \langle t, 0 \rangle$, for $0 \le t \le 1$.

I_2: $y = 1-x$, $0 \le x \le 1$. ~~So, $I_2 = \langle t, 1-t \rangle$, for $0 \le t \le 1$~~

at $t=0$, I_2 is $\langle 0,1 \rangle$
$t=1$, I_2 is $\langle 1,0 \rangle$

So, $\boxed{I_2 = \langle 1-t, t \rangle, \text{ for } 0 \le t \le 1.}$

I_3: $x = 0$, $0 \le y \le 1$, So, $I_3 = \langle 0, 1-t \rangle$, for $0 \le t \le 1$.

$I_i = \langle x(t), y(t) \rangle$

Next, $\int_C y^3 dx - x^3 dy = \int_{I_1 + I_2 + I_3}$

I_1: $\int_0^1 y^3 dx - x^3 dy$, $I_1 \Rightarrow \langle x(t), y(t) \rangle = \langle t, 0 \rangle$
$\Rightarrow \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \langle 1, 0 \rangle$

as t $\quad \frac{dx}{dt} dt \quad \frac{dy}{dt}$

$= \int_0^1 0 \cdot 1 dt - t^3 \cdot 0 dt = \boxed{0.}$

I_2: $\int_0^1 y^3 dx - x^3 dy$ $\quad I_2 \Rightarrow \langle x(t), y(t) \rangle = \langle 1-t, t \rangle$
$\left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \langle -1dt, dt \rangle$

$= \int_0^1 t^3(-1)dt - (1-t)^3 dt$

$= -\int_0^1 (t^3 - (1-t)^3) dt = -\left(\frac{1}{4} + \frac{1}{4}\right)$

$= \boxed{-\frac{1}{2}}$

I_3: $\int_0^1 y^3 dx - x^3 dy$ $\quad I_3$: $\langle x(t), y(t) \rangle = \langle 0, 1-t \rangle$
$\left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \langle 0, -dt \rangle$

$= \int_0^1 (1-t)^3 \cdot 0 dt - 0 = \boxed{0}$

Thus, $\int_C y^3 dx - x^3 dy = \boxed{-\frac{1}{2}}$

Keep in mind: 1. Parametrize your curve!
2. Convert everything to the variable t.
3. Integrate!

- More difficult parametrization
- Why does $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \Rightarrow$ conservative?