EL18 Lab 6 - live

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Given a function $f(x,y,z)$, max/min w.r.t. some additional restriction or constraint.

This constraint is given by $F(x,y,z) = 0$.

We seek $\boxed{\nabla f = \lambda \nabla F}$

λ → Lagrange multiplier!

Ex. Say the temperature at a point (x,y,z) is given by

$$T = 4xyz^2$$

Find the hottest point on the sphere $x^2+y^2+z^2 = 100$

hottest → maximize!

$x^2+y^2+z^2 = 100$: constraint! Lagrange!

So, $f(x,y,z) = 4xyz^2$

$F(x,y,z) = 0 = x^2+y^2+z^2-100$

$$\left.\begin{aligned} \nabla f &= (4yz^2, 4xz^2, 8xyz) \\ \nabla F &= (2x, 2y, 2z) \end{aligned}\right\} \quad \nabla f = \lambda \nabla F.$$

$$\begin{cases} 2\cancel{4}yz^2 = \lambda \cancel{2}x & \text{I} \\ 2\cancel{4}xz^2 = \lambda \cancel{2}y & \text{II} \\ 4\cancel{8}xyz = \lambda \cancel{2}z & \text{III} \end{cases}$$

(4) $x^2+y^2+z^2-100 = 0$ IV

First: Solve for $2z^2$ in I & II:

$$\frac{\lambda x}{y} = 2z^2 = \frac{\lambda y}{x}$$

$$\Rightarrow x^2 = y^2.$$

So, from III: $4xy\cancel{z} = \lambda \cancel{z}$

$$\lambda = 4xy.$$

So, $\boxed{2z^2} = \lambda \frac{x}{y} = (4x\cancel{y}) \cdot \frac{x}{\cancel{y}} = 4x^2 \quad (= \boxed{\cancel{4}2y^2})$

Now sub into eq$^=$ IV:

$$x^2+y^2+z^2-100 = 0$$

$$= y^2+y^2+z^2-100$$

$$= y^2+y^2+2y^2-100 = 0$$

$$\Rightarrow 100 = 4y^2$$

$$\hookrightarrow 25 = y^2 \Rightarrow y = \pm\sqrt{25} = \pm 5$$

$$\Rightarrow x^2 = y^2 = 25 \Rightarrow x = \pm 5.$$

Then, $z^2 = 2y^2 = 2 \cdot 25$

$$\hookrightarrow z = \pm\sqrt{50} = \pm\sqrt{2}\cdot 5$$

So, $(x,y,z) = (\pm 5, \pm 5, \pm\sqrt{2}\cdot 5)$. Now, go back and find the temp!

$$T = 4xyz^2 = 4(\pm 5)(\pm 5)(\pm\sqrt{2}\cdot 5)^2$$

$$= 5000$$

So, our hottest temp. is 5000 at points:

$(x,y,z) = (5, 5, \pm\sqrt{2}\,5)$

$(x,y,z) = (-5, -5, \pm\sqrt{2}\,5)$