EL06 Lab 6 - live

Monday, October 19, 2020 3:44 PM

- Lagrange multipliers

Sometimes, given a function $f(x,y,z)$ we wish minimize (or maximize) we are given an additional constraint.

Often, this constraint is given by $F(x,y,z) = 0$.

$$\nabla f = \lambda \nabla F$$

for some $\lambda \in \mathbb{R}$. λ is the lagrange multiplier.

Ex. Suppose temperature T at a point (x,y,z) is given by

$$T = 4xyz^2 = f(x,y,z)$$

Find the hottest point on the sphere $x^2+y^2+z^2 = 100$.

maximize! constraint! $\Rightarrow$ Lagrange!

$$F(x,y,z) = x^2+y^2+z^2-100 = 0$$

$$\nabla f = (4yz^2, 4xz^2, 8xyz)$$

$$\nabla F = (2x, 2y, 2z)$$

$$\nabla f = \lambda \nabla F$$

$$\nabla f \begin{cases} \cancel{2}\cancel{4}yz^2 = \cancel{2}\lambda x & \text{I} \\ \cancel{2}\cancel{4}xz^2 = \cancel{2}\lambda y & \text{II} \\ 4\cancel{8}xy\cancel{z} = \cancel{2}\lambda \cancel{z} & \text{III} \end{cases} \Bigg\} \lambda \nabla F$$

$$x^2+y^2+z^2 = 100 \quad \text{IV}$$

I & II:

$$\frac{\lambda x}{2y} = z^2 = \frac{\lambda y}{2x}$$

$$\boxed{x^2 = y^2} \qquad (\text{assuming } \lambda \neq 0)$$

III: $\lambda = 4xy$,

$$2z^2 = \frac{\lambda y}{x} = 4xy \cdot \frac{y}{x} = 4y^2$$

$$\boxed{z^2 = 2y^2}$$

Plug these into our constraint!

$$0 = x^2+y^2+z^2-100$$

$$= y^2+y^2+2y^2-100$$

$$\Rightarrow 100 = 4y^2$$

$$\Rightarrow 25 = y^2 \Rightarrow \boxed{y = \pm 5}$$

$$\Rightarrow \boxed{x = \pm 5}$$

Recall:

$$z^2 = 2y^2 = 50$$

$$\Rightarrow \boxed{z = \pm\sqrt{50} = \pm\sqrt{2}\,5}$$

So,

$$(x,y,z) = (\pm 5, \pm 5, \pm\sqrt{2}\,5)$$

So,

$$T = 4(\pm 5)(\pm 5)(\pm\sqrt{2}\cdot 5)^2$$

$$= 4 \cdot 25 \cdot (2 \cdot 25)$$

$$= \boxed{5000} \longrightarrow \text{max temp.!}$$

$z = \pm\sqrt{2}\,5$

and the points at which this occurs are:

$$\boxed{(x,y,z) = (5,5,\pm\sqrt{2}\,5)}$$

or

$$\boxed{(x,y,z) = (-5,-5,\pm\sqrt{2}\,5)}$$