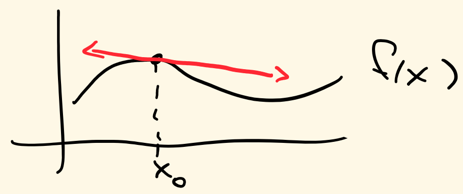
# EL18 Lab 3 - live

Tuesday, September 22, 2020 5:55 PM

Tangent Plane Approximation:

In 1-D:




$$f(x) \sim f'(x_0)(x - x_0)$$

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In 2-D: we have $f(x,y)$.

$$\Delta z \approx dz = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$= \underbrace{\frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)}_{\text{Tangent plane approx. of } f(x,y) \text{ @ } (x_0,y_0)}.$$

$$dz = \underbrace{\frac{\partial f}{\partial x}(x_0,y_0)\,dx + \frac{\partial f}{\partial y}(x_0,y_0)\,dy}_{\text{differential of } f(x,y) \text{ @ } (x_0,y_0)}$$

Ex. Let $f(x,y) = x^2 - 2y^2 + \tan(xy) + x^{-1}$

(i) Find the differential of $f(x,y)$.
(ii) Write the tangent plane approx. of $f(x,y)$ at $(x_0,y_0) = (-2, 0)$.

(i) $$\frac{\partial f}{\partial x} = 2x + \sec^2(xy)\cdot y - x^{-2},$$

$$\frac{\partial f}{\partial y} = -\underbrace{4y}_{0} + \underbrace{\sec^2(xy)}_{1}\,x$$

So, $$dz = \frac{\partial f}{\partial x}(x_0,y_0)\,dx + \frac{\partial f}{\partial y}(x_0,y_0)\,dy$$

$$= \left(2(-2) + -\frac{1}{(-2)^2}\right)dx$$

$$+ \left(0 + (1)\cdot(-2)\right)dy$$

$$\boxed{dz = -4.25\,dx - 2\,dy.}$$

(ii) $$dz \approx \Delta z = \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)$$

$$= -4.25(x-(-2)) + (-2)(y-0)$$

$$\boxed{\Delta z = -4.25(x+2) - 2y.}$$

$$\Delta z = z - z_0$$

$$z_0 = f(x_0,y_0)$$