EL18 Lab 1 - live

~~Monday, September 7, 2020 3:03 PM~~

Sept. 8, 2020
- Introduction
- Domain / range
- Level curves / graphs
- Limits, continuity.

Yurij

Domain: All values x such that $f(x)$ makes sense.

Ex: $f(x) = \sqrt{x}$ → If $x = 1$
$f(1) = \sqrt{1} = 1$ ✓
$f(2) = \sqrt{2}$ ✓

On the other hand, $x = -1$. Then $f(-1) = \sqrt{-1} = ?$

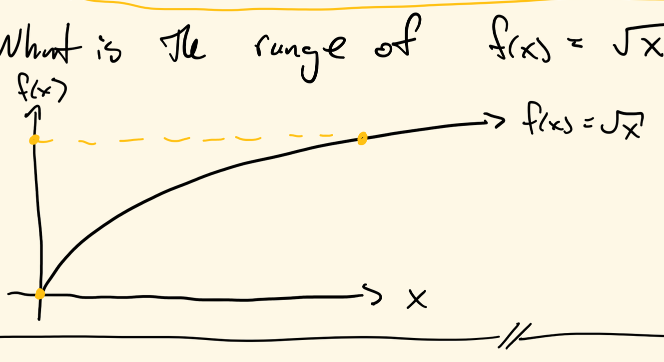
This means that $x = 1, x = 2$ belong to the domain of $f(x) = \sqrt{x}$, while $x = -1$ does not!

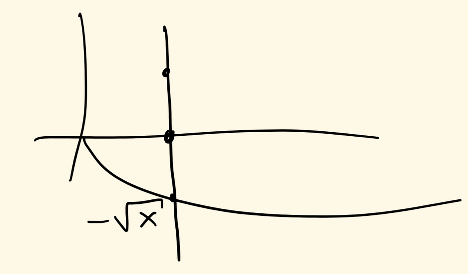
Q: What is the domain of $f(x) = \sqrt{x}$?

$\text{Dom}(f) = \boxed{\{x \in \mathbb{R} : x \geq 0\}}$

Range: The range is all values that the function can attain from the domain.

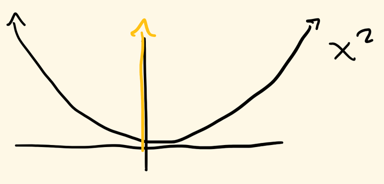
Q: What is the range of $f(x) = \sqrt{x}$?



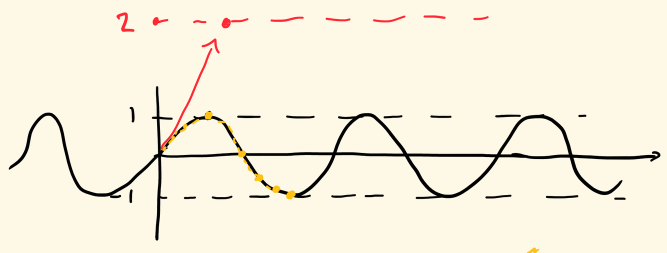


Ex.
1. $f(x) = x^2$
2. $f(x) = \ln(x)$
3. $f(x) = \sin(x)$
4. $f(x) = \tan(x)$.

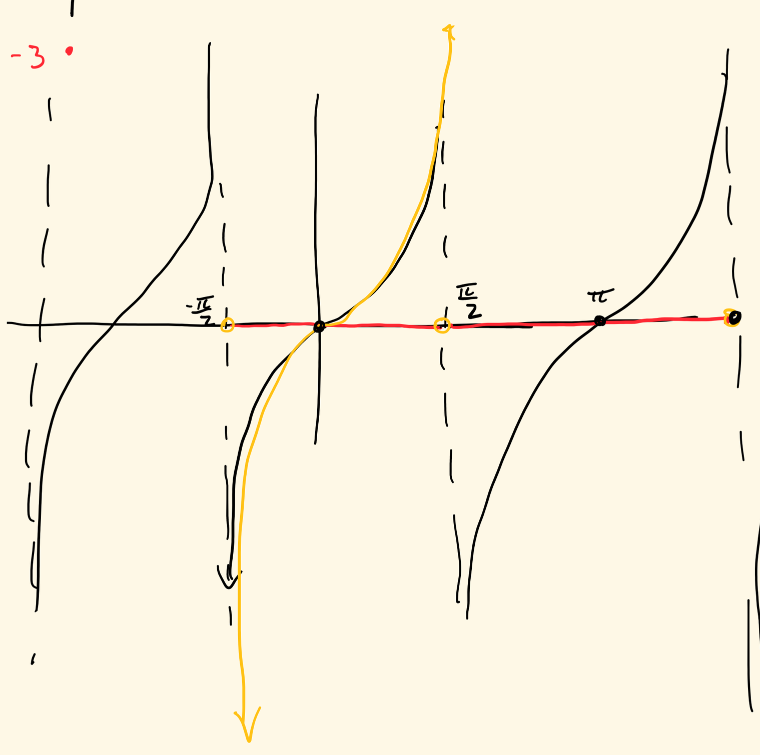
$x^2 = 4 \rightarrow +2, -2$

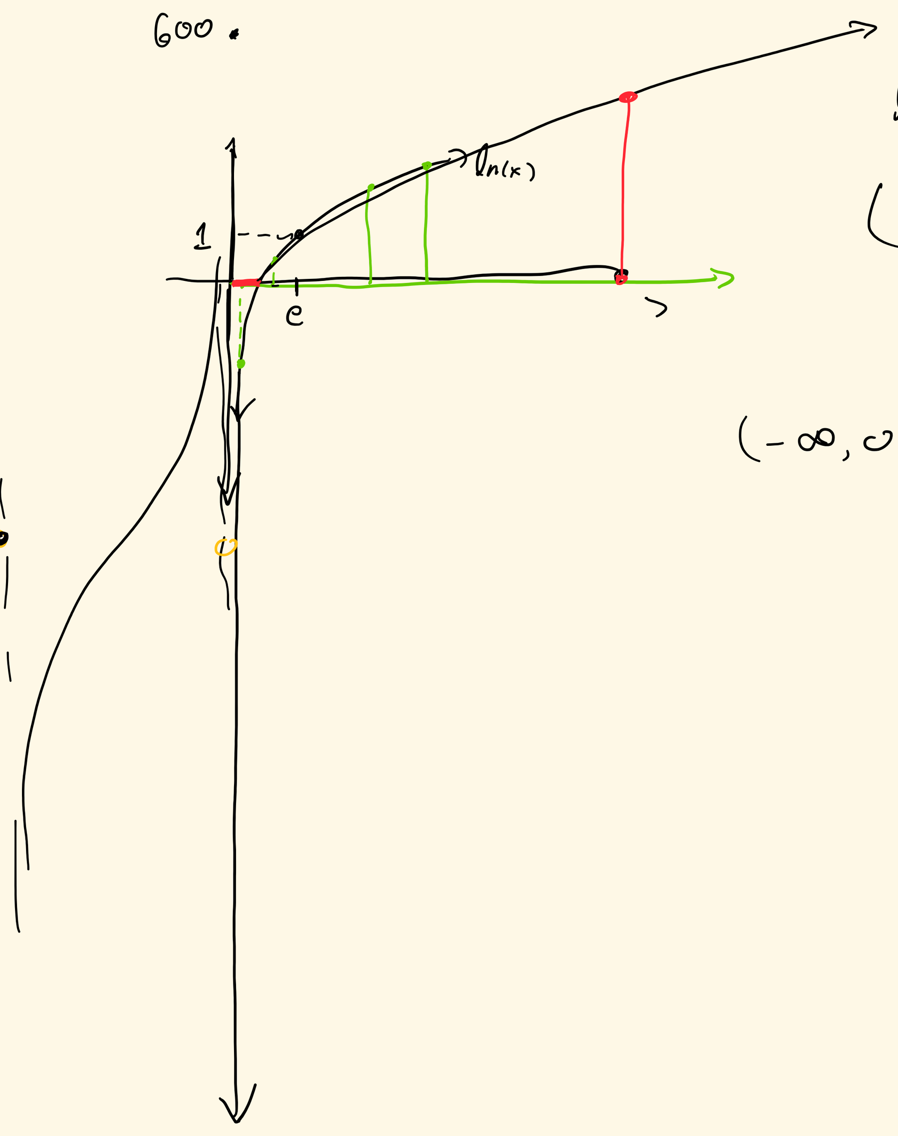


1. $\text{Dom}(x^2) = \mathbb{R} = \{x : -\infty < x < \infty\}$.
   $\text{Rng}(x^2) = [0, \infty)$
2. $\text{Dom}(\ln(x)) = (0, \infty)$
   $\text{Rng}(\ln(x)) = \boxed{(-\infty, \infty)}$
3. $\text{Dom}(\sin(x)) = (-\infty, \infty)$
   $\text{Rng}(\sin(x)) = [-1, 1]$



4. $\text{Dom}(\tan(x)) = \left(-\frac{(2n+1)}{2}, \frac{(2n+1)}{2}\right) \; \forall n \in \mathbb{N}$.
   $\text{Range}(\tan(x)) = (-\infty, \infty)$.
   → $\mathbb{R} - \left\{\left(\frac{2n+1}{2}\right)\pi\right\}, \; \forall n \in \mathbb{Z}$
   → $\mathbb{R} \setminus \left\{\frac{(2n+1)}{2}\pi\right\}, \; \forall n \in \mathbb{Z}$





$\lim_{x\to\infty}(\ln(x)) = \infty$

$\ln(e) = 1$
$x = e^{600}$
→ $\ln(x) = \ln(e^{600}) = \boxed{600}$

$(-\infty, 0)$

$f(x,y)$ with two inputs, we get a new value z, e.g. $f(x,y) = z$

Ex. $f(x,y) = x \ln(y^2 - x)$

$\text{Dom}_x = (-\infty, \infty)$

this to be positive!

$y^2 - x > 0$.

$\text{Dom}(f) = \{(x,y) \in \mathbb{R}^2 : y^2 > x\}$.

Ex. Find the domain & range of $g(x,y) = \sqrt{9 - x^2 - y^2}$

must be non-negative!

→ $9 - x^2 - y^2 \geq 0$

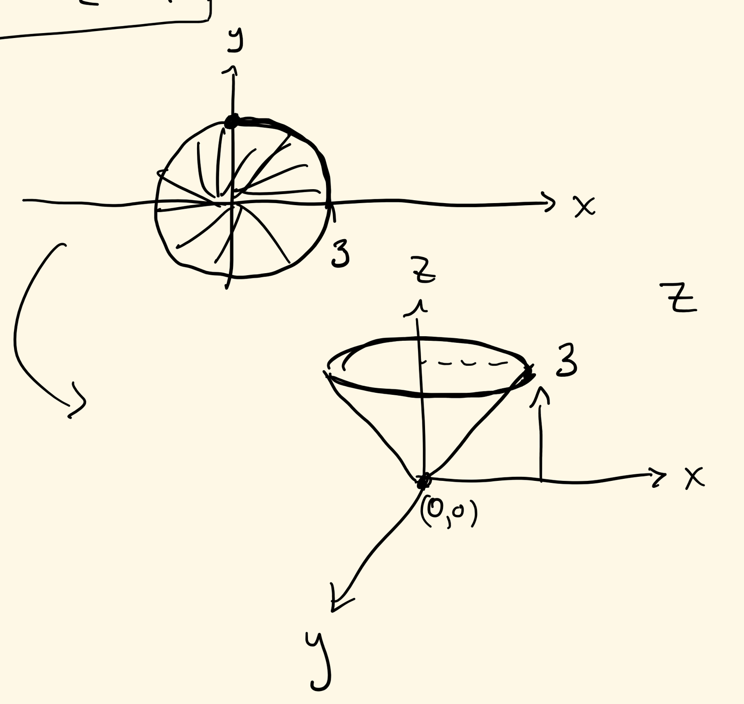
or $\boxed{x^2 + y^2 \leq 9}$

$x^2 + y^2 = 9 = 3^2$

What is the range ?

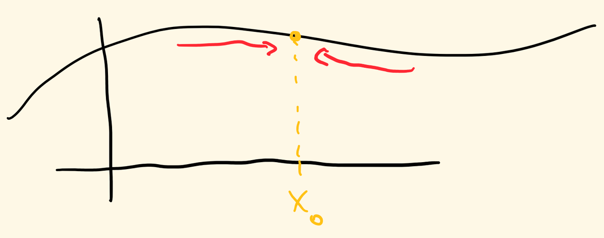
All values z such that $g(x,y) = z$ for some (x,y) in the domain.

→ $\text{Rng}(g(x,y)) = [0, 3]$.



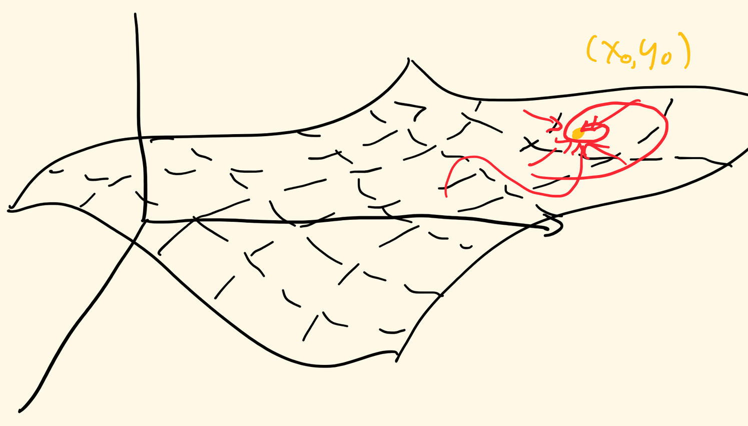
Limits: Given a function f, the limit at x_0 exists if
$$L = \lim_{x\to x_0} f(x)$$
makes sense.

In 1-D:



If $\lim_{x\to x_0^-} f(x) = \lim_{x\to x_0^+} f(x) = L$, then the limit exists and is exactly L.

In 2-D:



$f(x,y) = z$

In 2-D, the limit exists only if $\lim_{(x,y)\to(x_0,y_0)} f(x,y)$ is the same from any direction on any curve.

Ex. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ D.N.E.!

Set $y = 0$!

→ $\lim_{(x,0)\to(0,0)} \frac{x^2 - 0^2}{x^2 + 0^2} = \lim_{(x,0)\to(0,0)} 1 = 1.$

$y = mx$

$\neq$

Set $x = 0$!

$\lim_{(0,y)\to(0,0)} \frac{0^2 - y^2}{0^2 + y^2} = -1$

Therefore the limit as $(x,y) \to (0,0)$ D.N.E.!