EL06 Lab 2 - live

Monday, September 21, 2020 2:47 PM

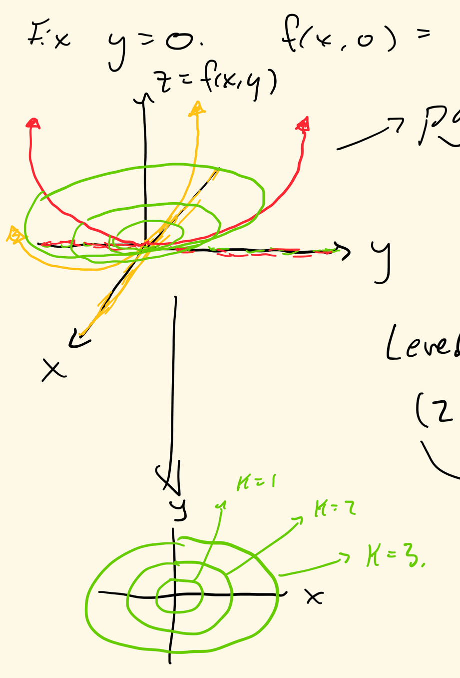
- Quiz 1 → $\boxed{\text{Sept. 28}}$!

Ex. Sketch the following curve:

$$f(x,y) = 4x^2 + 9y^2 = (2x)^2 + (3y)^2$$

First: $f(x,y) \geq 0$, $f = 0$ only at $(0,0)$.

Fix $y = 0$. $f(x,0) = 4x^2$, Fix $x = 0$, $f(0,y) = 9y^2$.



→ paraboloid

These are ellipses!!

Set $f(x,y) = K$, K is constant.

Level curves are of the form

$$\underbrace{(2x)^2 + (3y)^2 = k}_{\text{an ellipse!}}$$

Limits: In 2-D, we have too many directions!

General strategy:
1. Check two directions, see if they are the same.
   e.g. $x = 0$, go to work; $y = 0$, go to work.
2. Check all straight lines:
   e.g. set $y = mx$, take limit $x \to x_0$, if the limit does not depend on m, then maybe the limit exists.

If this all works out, it's time for polar coordinates!

Ex. Find $\lim_{(x,y)\to(0,0)} \frac{x^2y^3}{2x^2+3y^2}$.

Set $y = mx$, take lim as $x \to 0$.

So, $$\lim_{x\to 0} \frac{x^2(mx)^3}{2x^2+3(mx)^2} = \lim_{x\to 0}\frac{x^{\cancel{5}3} m^3}{\cancel{x^2}(2+3m^2)} = \lim_{x\to 0}\frac{x^3m^3}{2+3m^2} = 0$$ ✓ no m!

Now polar coordinates!

$x = r\cos\theta$, $y = r\sin\theta$

So, $$\lim_{r\to 0}\frac{(r\cos\theta)^2(r\sin\theta)^3}{2(r\cos\theta)^2+3(r\sin\theta)^2}$$
$$= \lim_{r\to 0}\frac{r^{\cancel{5}3}\cos^2\theta\sin^3\theta}{\cancel{r^2}(2\cos^2\theta+3\sin^2\theta)}$$

use that $\boxed{\cos^2\theta+\sin^2\theta = 1}$

$2\cos^2\theta + 2\sin^2\theta + \sin^2\theta$
$= 2(\cos^2\theta+\sin^2\theta) + \sin^2\theta$
$= 2 + \sin^2\theta$

$$= \lim_{r\to 0}\frac{r^3\cos^2\theta\sin^3\theta}{2+\sin^2\theta}.$$

No theta!

1. $\boxed{-1 \leq \cos^2\theta\sin^3\theta \leq 1}$
   $0 \leq \sin^2 \leq 1$, $-1 \leq \sin^3 \leq 1$
2. $\boxed{2 \leq 2+\sin^2\theta \leq 3}$
   $\frac{1}{3} \leq \frac{1}{2+\sin^2\theta} \leq \frac{1}{2}$

$2 + \sin^2\theta$, $0 \leq \sin^2\theta \leq 1$
$2 \leq 2+\sin^2\theta \leq 2+1 = 3$

$$-\frac{1}{2}r^3 \leq \frac{r^3\cos^2\theta\sin^3\theta}{2+\sin^2\theta} \leq \frac{1}{3}\cdot r^3$$

So, $$\lim_{r\to 0}\left(\frac{-1}{2}r^3\right) \leq \lim_{r\to 0}\frac{r^3\cos^2\theta\sin^3\theta}{(2+\sin^2\theta)} \leq \lim_{r\to 0}\left(\frac{1}{3}r^3\right)$$
$$0 \qquad = 0 \qquad 0$$

Therefore, $$\lim_{(x,y)\to(0,0)}\frac{x^2y^3}{2x^2+3y^2} = 0 \;!$$

$f(x,y)$

$-17|r^3| \leq 17r^3 \leq 17|r^3|$ $\quad r \to 0$
$\downarrow \qquad \downarrow \qquad \downarrow$
$0 \qquad 0 \qquad 0$

$2 \leq 2+\sin^2\theta \leq 3$

We need $\frac{1}{2+\sin^2\theta}$.

$\frac{2}{1} \leq \frac{2+\sin^2\theta}{1}$ $\quad$ $\frac{1}{2+\sin^2\theta} \leq \frac{1}{2}$

$\frac{2+\sin^2\theta}{1} \leq \frac{3}{1}$ $\quad$ $\frac{1}{3} \leq \frac{1}{2+\sin^2\theta}$

$$\boxed{\frac{1}{3} \leq \frac{1}{2+\sin^2\theta} \leq \frac{1}{2}}$$

$-\frac{1}{3} \geq -\frac{1}{2+\sin^2\theta} \geq -\frac{1}{2}$

$0.33\ldots \leq \quad \leq 0.5$

$-0.5 \leq -0.33$

Partial Derivatives:

Ex. Find $\frac{\partial f}{\partial y}$ of $f(x,y) = x^2+y^2$

$$\frac{\partial}{\partial y}(x^2+y^2) = 0 + 2y.$$

Ex. Find $\frac{\partial f}{\partial y}$ of $f(x,y) = x^2+xy+y^2$

$$\frac{\partial}{\partial y}(x^2+xy+y^2)$$

0; $= x\frac{\partial}{\partial y}(y) = x$; $2y$

So, $$\frac{\partial f}{\partial y} = x + 2y.$$

Ex. Find $\frac{\partial f}{\partial x}$ of $f(x,y) = x^2e^{xy} + \sin(ax)y$

$$\frac{\partial}{\partial x}\left(x^2e^{xy} + \sin(ax)\cdot y\right)$$

$$\frac{\partial}{\partial x}(x^2e^{xy}) = \frac{\partial}{\partial x}(x^2)\cdot e^{xy} + x^2\frac{\partial}{\partial x}(e^{xy})$$
$$= 2xe^{xy} + x^2e^{xy}\cdot\frac{\partial}{\partial x}(xy)$$
$$= 2xe^{xy} + x^2e^{xy}\cdot y.$$

$$\frac{\partial}{\partial x}(\sin(ax)\cdot y) = y\frac{\partial}{\partial x}(\sin(ax))$$
$$= y\cos(ax)\cdot a$$

So, $$\boxed{\frac{\partial f}{\partial x} = 2xe^{xy} + x^2ye^{xy} + ay\cos(ax)}$$

Ex. Check if $u(x,y) = x^2\sin(y) + y^2\sin(x)$ satisfies $u_x + u_{xy} = 2x(\boxed{\sin(x)} + \sin(y)) + y(y+2)\cos(x)$.

So, $$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(x^2\sin(y) + y^2\sin(x)\right)$$
$$= \boxed{2x\sin(y) + y^2\cos(x).}$$

$$u_{xy} = \frac{\partial}{\partial y}\left(2x\sin(y) + y^2\cos(x)\right)$$
$$= 2x\cos(y) + 2y\cos(x).$$

So, $$u_x + u_{xy} = 2x\sin(y) + y^2\cos(x) + 2x\cos(y) + 2y\cos(x)$$
$$= 2x(\sin(y) + \boxed{\cos(y)}) + \cos(x)\,y(y+2)$$

✗ Not the same!

Therefore, $u(x,y)$ does not satisfy the given equation.