EL06 Lab 1 - live

Monday, September 14, 2020 11:20 AM

Sept. 14, 2020
- Introduction
- Domain / range
- level curves / graphs
- limits, continuity

The domain of a function $f(x)$ is the set of values you can "legally" put into the function and get something back that makes sense.

Ex.) $f(x) = \sqrt{x}$. If $x=1$, $\sqrt{1} = 1$ ✓
$x = 2$, $\sqrt{2}$

But, if $x=-1$, then $\sqrt{-1} = ?$ ✗

This means $x = 1, 2$ are in the domain, while $x = -1$ is not.

Q: What is the domain of $\sqrt{x}$?

$\mathrm{Dom}(\sqrt{x}) = [0, \infty)$.

The range of a function $f(x)$ is everywhere that $f(x)$ can "get to" or "attain" from its domain.

Ex. Is 1 in the range of $f(x) = \sqrt{x}$?

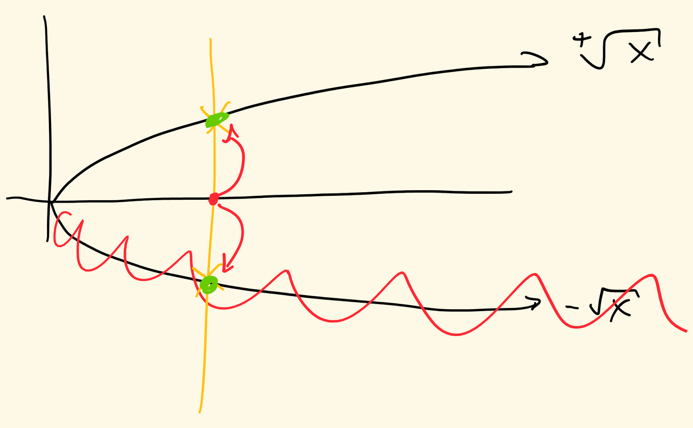
Yes! Since $f(1) = \sqrt{1} = 1$ ✓

Is -1 in the range? No!

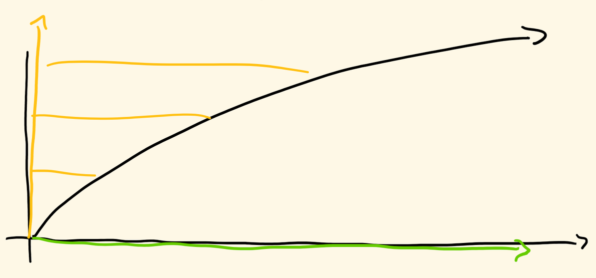
Since $f(x) \geq 0$ for all x in the domain.

$x^2 = 1$ has $x = \pm 1$

$\hookrightarrow y = \sqrt{x} = f(x)$



Q: What is the range of $f(x) = \sqrt{x}$?



$\mathrm{Range}(f(x)) = [0, \infty)$

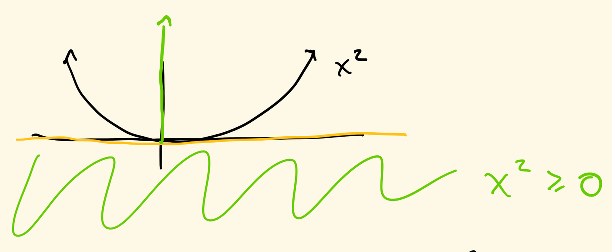
Ex. Find the dom / range of the following:

a.) $f(x) = x^2$
b.) $f(x) = \ln(x)$
c.) $f(x) = \sin(x)$
d.) $f(x) = \tan(x)$.

a.) $f(x) = x^2$ $\mathrm{Dom}(f) = \{x : x \in \mathbb{R}\}$.

$\mathrm{Rng}(f) = \{x \in \mathbb{R} : x \geq 0\}$

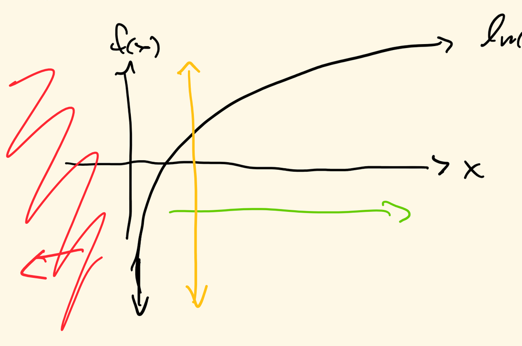
$= \{y \in \mathbb{R} : y \geq 0\}$



b.) $f(x) = \ln(x)$ $\mathrm{Dom}(f) = (0, \infty)$

$= \{x \in \mathbb{R} : x > 0\}$

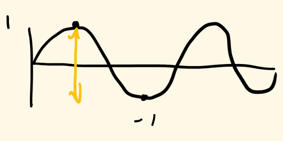
$\mathrm{Rng}(f) = (-\infty, \infty)$



c.) $f(x) = \sin(x)$, $\mathrm{Dom}(f) = (-\infty, \infty)$

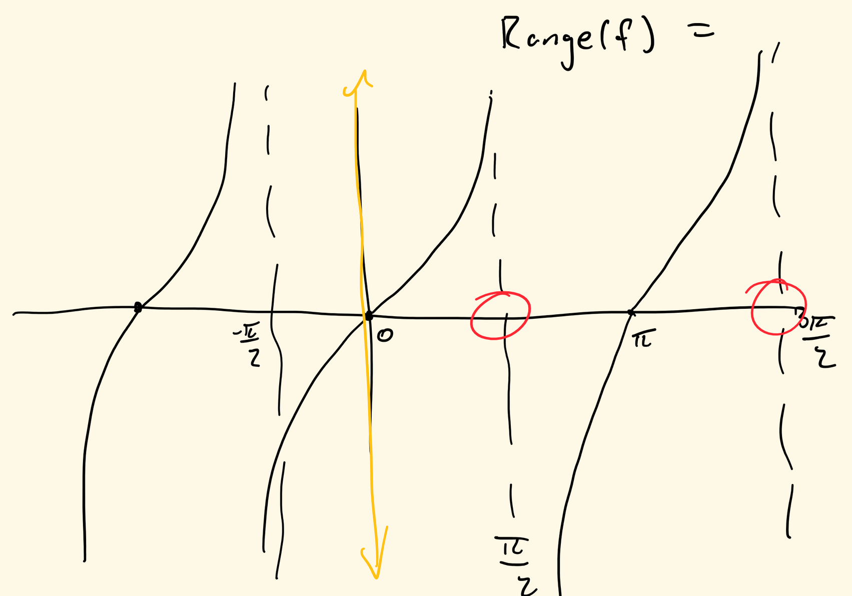
$= \mathbb{R}$

$\mathrm{Rng}(f) = \cancel{(-1, 1)}$ or $[-1, 1]$ ✓



d.) $f(x) = \tan(x)$, $\mathrm{Dom}(f) =$

$\mathrm{Range}(f) =$



$\frac{\pi}{2} \notin$ ~~Range~~ Domain

$\frac{3\pi}{2} \notin$ Domain

$\vdots$

$\frac{5\pi}{2}$

$\frac{7\pi}{2}$

$\vdots$

$\frac{(2n+1)}{2}\pi$ for $n \in \mathbb{Z}$

$\boxed{n\pi + \frac{\pi}{2}}$

$\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$

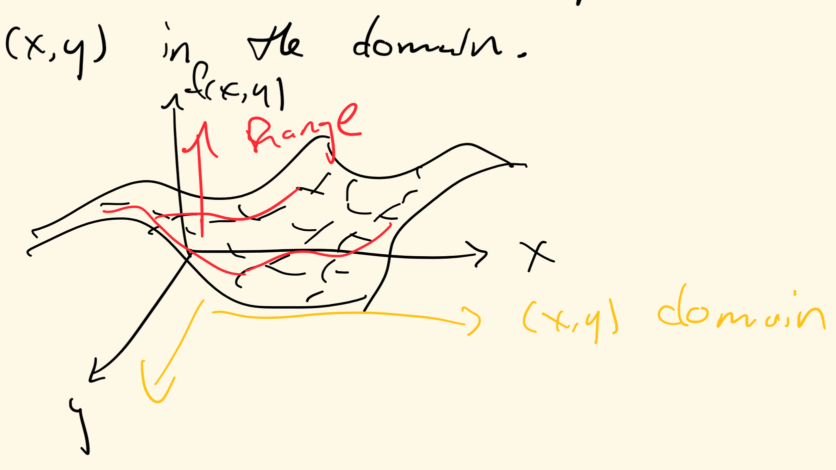
$\mathrm{Dom}(f) = \{x \in \mathbb{R} : x \neq n\pi + \frac{\pi}{2}\ \forall n \in \mathbb{Z}\}$

$\mathrm{Range}(f) = \{b \in \mathbb{R}\}$.

The same idea applies to the domain & range of a multivariable function $f(x,y)$!

Domain is all values (x,y) that can legally be input to $f(x,y)$.

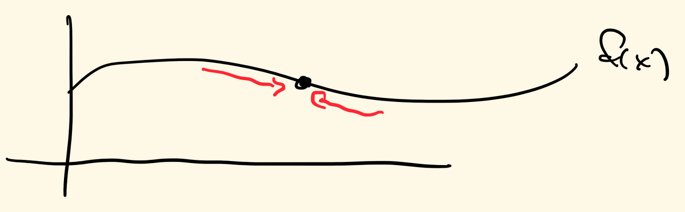
Range is all values z such that $f(x,y)$ reaches z for some value (x,y) in the domain.



Ex. $\underbrace{x}_{\in \mathbb{R}}\ln(\underbrace{y^2 - x}_{> 0}) = f(x,y) \Rightarrow$ Find the domain

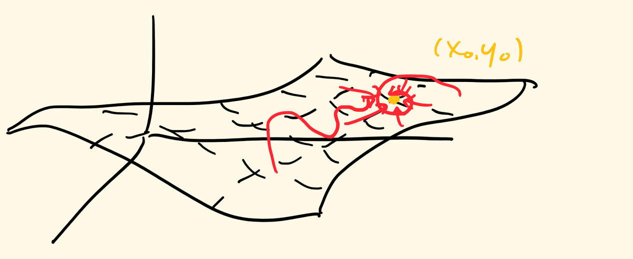
Ex. $\sqrt{\underbrace{9 - x^2 - y^2}_{\geq 0}} = g(x,y) \Rightarrow$ Find the dom & range.

Limits: In 1-D, we usually take "left" & "right" limits: $\lim_{x \to x_0^-} f(x)$, $\lim_{x \to x_0^+} f(x)$, if they are the same, we say that $\lim_{x \to x_0} f(x) = L$.



two directions.

In 2-D:



We need all limits to be the same from every possible direction!

Ex. Show that $\lim_{(x,y) \to (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ D.N.E.!

Choose the line $x = 0$.

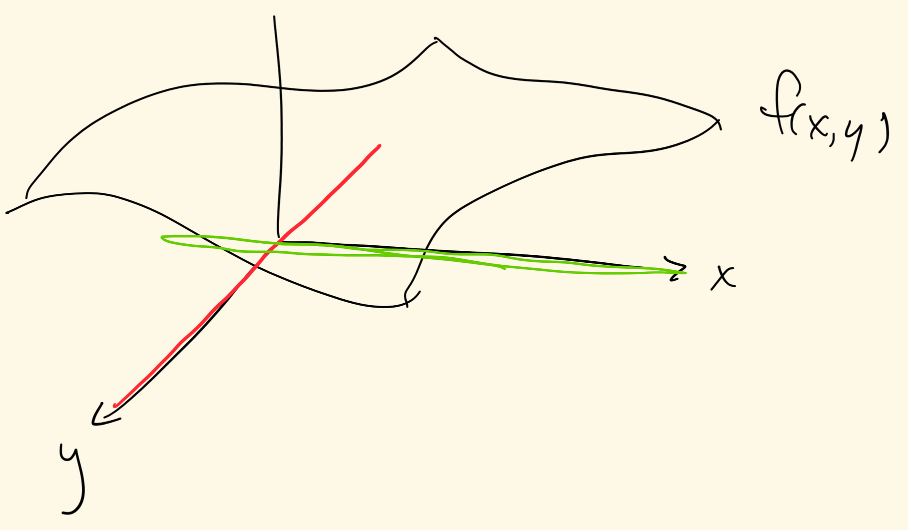
Then, $\lim_{(y \to 0)} \frac{0^2 - y^2}{0^2 + y^2} = -1$

Choose the line $y = 0$.

Then, $\lim_{x \to 0} \frac{x^2 - 0^2}{x^2 + 0^2} = +1$

$\neq$

Therefore the limit D.N.E.



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1. Check lines $x = 0$, $y = 0$.
2. Check lines $y = mx$ or $y = x^p$, for some $p > 0$.
3. Try $x = r\cos\theta$, $y = r\sin\theta$ as $(x,y) \to (0,0)$.

Continuity: In 2-D, if $\lim_{(x,y) \to (x_0, y_0)} f(x,y)$ exists, we say $f(x,y)$ is continuous at (x_0, y_0).

Then, $f(x,y)$ is continuous on some set $D \subset \mathbb{R}^2$ if $\lim_{(x,y) \to (x_0, y_0)} f(x,y)$ exists for every pair $(x_0, y_0) \in D$.

Otherwise $f(x,y)$ is discontinuous at (x_0, y_0).