Lab 8 Live

Thursday, May 28, 2020 8:05 PM

Ex. $\int_0^1 \left(\int_x^{2x} \left(\int_0^y 2xyz \, dz \right) dy \right) dx$

$= xyz^2 \Big|_{z=0}^{z=y}$

$= xy^3$

$\int_0^1 \left(\int_x^{2x} xy^3 \, dy \right) dx$

$= x\frac{y^4}{4} \Big|_{y=x}^{y=2x}$

$= \frac{x}{4}\left((2x)^4 - x^4 \right)$

$= \frac{x^5}{4}(15) = \frac{15x^5}{4}.$

$\int_0^1 \frac{15x^5}{4} dx = \frac{15}{4} \cdot \frac{1}{6} \cdot x^6 \Big|_0^1$

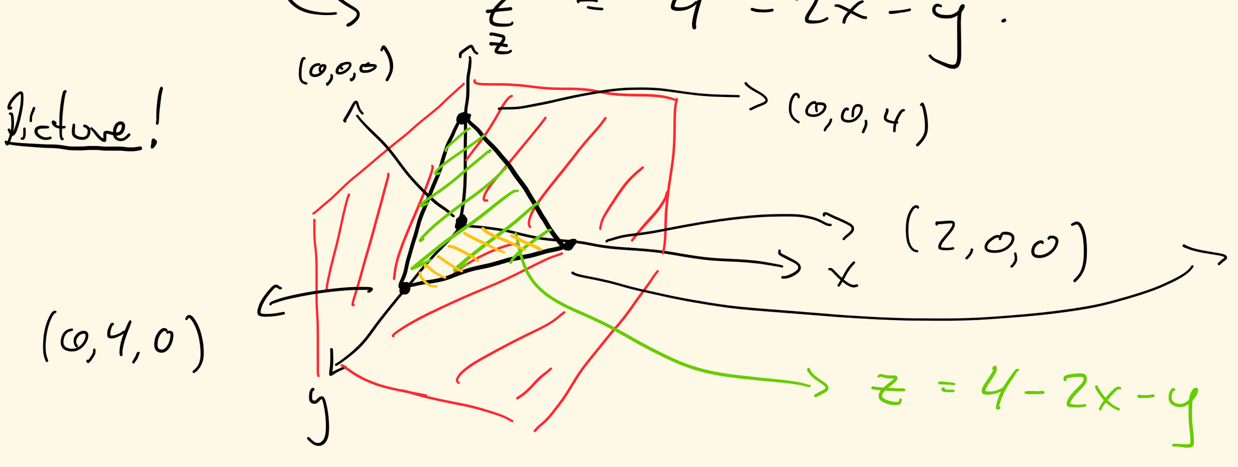
$= \frac{15}{24} = \boxed{\frac{5}{8}}$

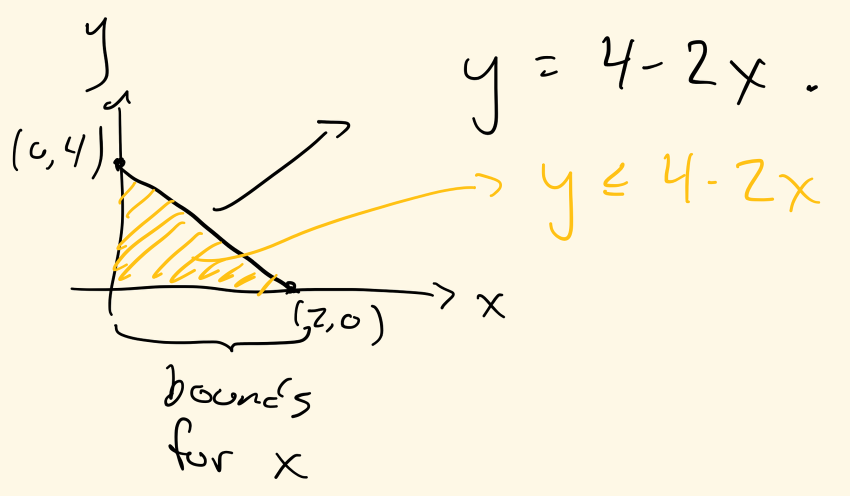
Ex. Find the volume of the tetrahedron enclosed by the coordinate planes & the plane

$2x + y + z = 4$

$z = 4 - 2x - y.$

Picture!





when $(x,y) = 0$, $z = 4$

when $(x,z) = 0$, $y = 4$

when $(y,z) = 0$, $x = 2$

$R = \{(x,y,z) : 0 \le x \le 2,\ 0 \le y \le 4-2x,\ 0 \le z \le 4-2x-y\}.$

$\text{Volume} = \iiint_R dV = \int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} dz\, dy\, dx$

$= \int_0^2 \int_0^{4-2x} (4-2x-y)\, dy\, dx$

$= \int_0^2 \left[(4-2x)y - \frac{y^2}{2} \right] \Big|_{y=0}^{y=4-2x} dx$

$= \int_0^2 \left[(4-2x)(4-2x) - \frac{1}{2}(4-2x)^2 \right] dx$

$= \frac{1}{2} \int_0^2 (4-2x)^2 dx \qquad , \quad u = 4-2x, \quad -\frac{1}{2} du = dx$

$= -\frac{1}{4} \int_0^2 u^2 du$

$= -\frac{1}{4} \cdot \frac{1}{3} \cdot u^3 \Big|_{x=0}^{x=2}$

$= -\frac{1}{12} \cdot (4-2x)^3 \Big|_{x=0}^{x=2}$

$= 0 - \left(-\frac{1}{12}\right)(4^3)$

$= \frac{4^3}{12} = \boxed{\frac{16}{3}}$

Ex. Use cyl. coords. to find the vol. of the region lying in the first octant, bounded above by

$z = 4 - x^2 - y^2$

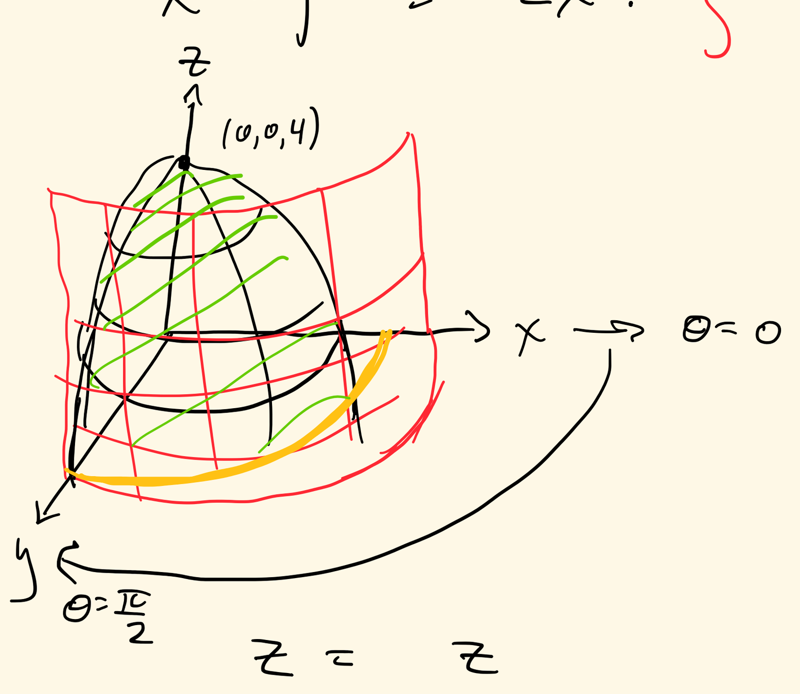
and inside

$x^2 + y^2 = 2x.$ $\rightarrow$ $x^2 + y^2 \le 2x$

$r^2 \le 2x = 2r\cos\theta$

$0 \le r \le 2\cos\theta$

Picture!



remember:

$z = z$

$x = r\cos\theta$, $y = r\sin\theta$ $\Big\}$ $x^2 + y^2 = r^2$

$E = \begin{cases} 0 \le z \le 4 - x^2 - y^2 = 4 - r^2. \\ 0 \le r \le 2\cos\theta \\ 0 \le \theta \le \frac{\pi}{2} \end{cases}$

$\boxed{dx\,dy = r\,dr\,d\theta}$

$\text{Volume} = \iiint_E dV$

$= \int_0^{\frac{\pi}{2}} \int_0^{2\cos\theta} \int_0^{4-r^2} dz\, r\, dr\, d\theta$

$= \int_0^{\frac{\pi}{2}} \int_0^{2\cos\theta} r(4-r^2)\, dr\, d\theta$

$= \int_0^{\frac{\pi}{2}} \left(2r^2 - \frac{r^4}{4} \right) \Big|_{r=0}^{r=2\cos\theta} d\theta$

$= \int_0^{\frac{\pi}{2}} \left(2(2\cos\theta)^2 - \frac{(2\cos\theta)^4}{4} \right) d\theta$

Use $\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$

$= \boxed{\frac{5\pi}{4}}$