Lab 1 Live

Tuesday, May 5, 2020 7:46 PM

Welcome! I'll wait until 8:10 to start.

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→ ask questions!

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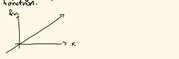
→ notes will be here!

- Domain/range of functions
- Level curves / graphs of functions
- Limits & continuity.

Some informal definitions: The domain of a function is all values you can possibly input into the function.

Ex: $f(x) = x$.



any real number is a legal input!

Therefore, the domain, $\text{Dom}(f)$, is all real numbers!

i.e. $\text{Dom}(f) = \mathbb{R} = (-\infty, \infty)$ or $= \{-\infty < x < \infty\}$

Ex. $f(x) = \sqrt{x}$. $\text{Dom}(f) = x \geq 0$

$f(0) = \sqrt{0} = 0$.

$f(a) = \sqrt{a}$, makes sense if $a > 0$.

If $x < 0$, $f(x) = \sqrt{x} = \sqrt{-1 \cdot |x|} = \sqrt{-1}\sqrt{|x|}$

↳ this is undefined (in this course).

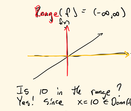
So, $\text{Dom}(f) = x \geq 0$, $[0, \infty)$, $0 \leq x < \infty$

On the other hand, all values that a function can attain from its domain is its range of $f(x)$.

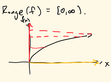
Ex. $f(x) = x$ $\quad$ $f(x) = \sqrt{x}$

$\text{Dom}(f) = \mathbb{R} = (-\infty, \infty)$ $\quad$ $\text{Dom}(f) = [0, \infty)$.

$\text{Range}(f) = (-\infty, \infty)$ $\quad$ $\text{Range}(f) = [0, \infty)$.



Is 10 in the range? Yes! Since $x \in \text{Dom}(f)$, $f(x) = f(10) = 10$



Is 10 in the range? Yes! Since $x \geq 0$ (with domain!), then $f(x) = f(100) = \sqrt{100} = 10$.

The same idea applies to a multivariable function!

Given $(x_1, x_2, \cdots, x_n)$, $f(x_1, x_2, x_3, \cdots, x_n)$ is a function that assigns each "n-tuple" to a value.

Let's focus on 2-D. Given a function $f(x,y)$ and an input (x,y), the domain is all values $(x,y) \in \mathbb{R}^2$ such that $f(x,y)$ makes sense. (All "legal" inputs).

The range of $f(x,y)$ is all values that $f(x,y)$ can reach given $(x,y) \in \text{Dom}(f)$.

Ex. $f(x,y) = x \ln(y^2 - x)$

$\text{Dom}(f) = \{(x,y) \in \mathbb{R}^2 : y^2 > x\}$.

x can be anything

must be positive!!

$\Rightarrow y^2 > x$

$g(x,y) = \sqrt{9 - x^2 - y^2}$ → must be non-negative!

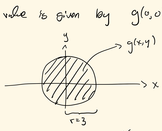
$\text{Dom}(g) = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 9\}$.

$9 - x^2 - y^2 \geq 0$ (this is $g(x,y)$)

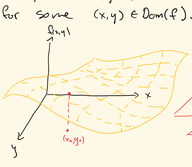
$\text{Range}(g) = [0, 3]$

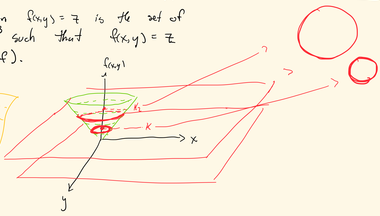
$g(x,y) \geqslant 0$ for $(x,y) \in \text{Dom}(g)$.

Largest value is given by $g(0,0) = \sqrt{9} = 3$.



The GRAPH of a function $f(x,y) = z$ is the set of all points $(x,y,z) \in \mathbb{R}^3$ such that $f(x,y) = z$ for some $(x,y) \in \text{Dom}(f)$.

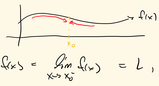




From here, we can talk about LEVEL SETS / CURVES

The idea is to fix a value $K \in \mathbb{R}$ and look at all the values $(x,y) \in \text{Dom}(f)$ such that $f(x,y) = K$.

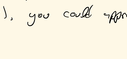
Limits! In one dimension, there are two directions to approach a point from:



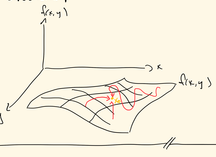
If $\lim_{x \to x_0^-} f(x) = \lim_{x \to x_0^+} f(x) = L$,

then $\lim_{x \to x_0} f(x) = L$.

When such a limit exists, $f(x)$ is continuous at x_0.



In 2-D (or higher), you could approach from many directions!



Ex. Show that $\lim_{(x,y) \to (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ DNE!

↓

Show that the limit has different values from different directions!

Proof: Approach along $x = 0$.

$$\lim_{(x,y) \to (0,0)} \frac{-y^2}{y^2} = -1$$

Approach along $y = 0$

$$\lim_{(x,y) \to (0,0)} \frac{x^2}{x^2} = 1$$

Not the same!

↓

limit DNE!

Ex. Does $\lim_{(x,y) \to (0,0)} \frac{xy^2}{x^2 + y^4}$ exist?

Set $y = mx$, for m arbitrary.

$$\lim_{(x,y) \to (0,0)} \frac{x(mx)^2}{x^2 + (mx)^4} = \lim_{(x,y) \to (0,0)} \frac{m^2 x^3}{x^2(1 + m^4 x^2)}$$

$$= \lim_{(x,y) \to (0,0)} \frac{mx}{1 + m^4 x^2}$$

$$= 0,$$

This is true for any m!

Therefore, $\lim_{(x,y) \to (0,0)} \frac{xy^2}{x^2 + y^4} = 0$.