

## Extra Practice Problems

MATH201 - Fall, 2019

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Special thanks (again) to Carlos Contreras for providing some of these practice problems.

In the following, there are questions concerning Fourier series, eigenvalue problems, the heat equation and the wave equation. One should also review material that came before these topics, and I would suggest going through old assignment questions, midterms as well as the extra problems I have posted already. Even if you've done them before, it may be useful to do them again without looking at your notes to challenge yourself. Good luck!

### 1. FOURIER SERIES

1. Compute the Fourier series of the following function on the given interval:

$$f(x) = x, \quad -\pi < x < \pi.$$

2. Compute the Fourier **sine** and **cosine** series of the following function on the given interval:

$$f(x) = x - x^2, \quad 0 < x < 1.$$

3. Compute the Fourier cosine series of the following functions on the given interval:

a.)  $f(x) = \pi - x, \quad 0 < x < \pi$

b.)  $f(x) = e^{-x}, \quad 0 < x < 1$

c.)  $f(x) = \sin(x), \quad 0 < x < \pi$

4. Compute the Fourier sine series of the following functions on the given interval:

a.)  $f(x) = -1, \quad 0 < x < 1$

b.)  $f(x) = x^2, \quad 0 < x < \pi$

c.)  $f(x) = \cos(x), \quad 0 < x < \pi$

### 2. EIGENVALUE PROBLEMS

5. Find all nontrivial solutions to the eigenvalue problem

$$y'' - \lambda y = 0, \quad 0 < x < L, \quad y(0) = 0, \quad y(L) = 0.$$

6. Find all nontrivial solutions to the eigenvalue problem

$$y'' + \lambda y = 0, \quad 0 < x < L, \quad y'(0) = 0, \quad y'(L) = 0.$$

7. Find all nontrivial solutions to the eigenvalue problem

$$y'' + \lambda y = 0, \quad 0 < x < L, \quad y(0) - y'(0) = 0, \quad y(L) = 0.$$

8. Find all nontrivial solutions to the eigenvalue problem

$$y'' + 4y' + \lambda y = 0, \quad 0 < x < \frac{\pi}{2}, \quad y(0) = 0, \quad y(\pi/2) = 0.$$

## 3. HEAT EQUATION

9. Solve the following heat equation:

$$\begin{cases} u_t = \alpha u_{xx}, & \alpha > 0, & 0 < x < 1, & t > 0, \\ u(0, t) = u(1, t) = 0, & & t > 0, \\ u(x, 0) = x(1 - x), & & 0 < x < 1. \end{cases}$$

10. Solve the following heat equation:

$$\begin{cases} u_t = \alpha u_{xx}, & \alpha > 0, & 0 < x < 1, & t > 0, \\ u_x(0, t) = u_x(1, t) = 0, & & t > 0, \\ u(x, 0) = x(1 - x), & & 0 < x < 1. \end{cases}$$

11. Solve the following heat equation:

$$\begin{cases} u_t = u_{xx}, & 0 < x < \pi, & t > 0, \\ u(0, t) = 0, & u(\pi, t) = 3\pi, & t > 0, \\ u(x, 0) = 0, & & 0 < x < \pi. \end{cases}$$

12. Find a formal solution to the initial value problem

$$\begin{cases} u_t = u_{xx} + 6x - 2, & 0 < x < 1, & t > 0, \\ u(0, t) = 0, & u(1, t) = -1, & t > 0, \\ u(x, 0) = -x^3, & & 0 < x < 1. \end{cases}$$

## 4. WAVE EQUATION

13. Solve the following wave equation:

$$\begin{cases} u_{tt} = 4u_{xx}, & 0 < x < \pi, & t > 0, \\ u(0, t) = u(\pi, t) = 0, & & t > 0, \\ u(x, 0) = x^2(\pi - x), & & 0 < x < \pi, \\ u_t(x, 0) = 0, & & 0 < x < \pi. \end{cases}$$

14. Solve the following wave equation:

$$\begin{cases} u_{tt} = 9u_{xx}, & 0 < x < \pi, & t > 0, \\ u(0, t) = u(\pi, t) = 0, & & t > 0, \\ u(x, 0) = \sin(4x) + 7\sin(5x), & & 0 < x < \pi, \\ u_t(x, 0) = \begin{cases} x, & 0 < x < \pi/2, \\ \pi - x, & \pi/2 < x < \pi. \end{cases} \end{cases}$$